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STUDIES OF TARGET DETECTION ALGORITHMS  
WHICH USE POLARIMETRIC RADAR DATA

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# Studies of Target Detection Algorithms Which Use Polarimetric Radar Data\*

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## ABSTRACT

This paper presents the results of a study of target detection algorithms which use polarimetric radar data. Improved polarimetric target and ground clutter models are presented and the performance of algorithms using these models is derived. Theoretical performance predictions based on typical polarimetric target and clutter parameters are presented and a comparison of the performance of various algorithms is given.

## INTRODUCTION

This paper considers the use of polarimetric radar information in the detection and discrimination of targets embedded in a ground clutter background. A frequency diverse radar such as the HOWLS [1] radar is adopted as a baseline. The basic radar resolution is assumed to be on the order of the size of a typical target (10 m by 10 m). We assume the frequency diverse radar has a fully polarimetric measurement capability. To measure the full polarization scattering matrix (PSM), the radar transmits two orthogonal linear polarizations at each frequency. First, horizontal polarization is transmitted and the radar receives two linear orthogonal components (denoted HH and HV). Next, vertical polarization is transmitted and the radar receives the VV and VH returns. By reciprocity, we assume VH=HV and thus the HH, HV, and VV returns comprise the total information contained in the polarization scattering matrix. Frequency diversity is used to obtain successively independent scattering matrix measurements for multi-look polarimetric processing schemes.

In the following section of the paper, we introduce the basic polarimetric measurement model and present a statistical description of both targets and clutter. In the remaining sections of the paper, we apply these statistical models of targets and clutter and derive the optimal polarimetric detector (OPD). We show the fundamental structure of the algorithm which processes polarimetric measurement data in an optimal manner. We also derive the best linear polarimetric detector, the polarimetric matched filter (PMF), and relate the structure of this processing scheme to simple

polarimetric target types. Next, we propose more realistic product model characterizations of targets and clutter which account for the effects of spatial non-homogeneity of ground clutter and aspect angle variability of real targets. These new polarimetric target and clutter models are used to evaluate the sensitivity of the OPD and PMF detectors to the effects of spatial variability of ground clutter and aspect angle variability of targets. We compare the performance of these algorithms to simpler detectors which use only amplitude information to detect targets. Theoretical performance predictions and algorithm performance comparisons are presented in terms of detector ROC curves (receiver operating characteristics) which show detection probability versus false alarm probability for various polarimetric processors. Performance results are presented parametrically as a function of target-to-clutter ratio, number of independent polarimetric measurements processed, and detector type. The ability to discriminate between target types using differences in polarimetric scattering properties is also investigated. The final section of the paper summarizes our findings and describes possible extensions and future studies.

## THE BASIC POLARIMETRIC MEASUREMENT MODEL

We begin by describing the basic mathematical modeling of targets and clutter used in our studies. These models are then used to derive the optimal polarimetric detector and the polarimetric matched filter. We write the radar return as the polarimetric feature vector  $\underline{X}$ , where

$$\underline{X} = \begin{bmatrix} HH_i + jHH_q \\ HV_i + jHV_q \\ VV_i + jVV_q \end{bmatrix} = \begin{bmatrix} HH \\ HV \\ VV \end{bmatrix} \quad (1)$$

The elements of the vector, HH, HV, and VV are appropriately modeled as having a complex Gaussian probability density function (PDF). This PDF is given by the following expression

$$f(\underline{X}) = \frac{1}{\pi^n |\Sigma|} \exp -\underline{X}^\dagger \Sigma^{-1} \underline{X} \quad (2)$$

where  $\Sigma = E\{\underline{X} \underline{X}^\dagger\}$  is the covariance of the polarimetric feature vector. Note that the data has

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zero mean ( $E\{X\} = 0$ ). Thus, the complete characterization of the jointly Gaussian returns HH, HV, and VV is given in terms of an appropriate covariance matrix,  $\Sigma$ . The corresponding covariance matrices which we use for target and clutter data (in a linear polarization basis) are of the form

$$\Sigma = \sigma \begin{bmatrix} 1 & 0 & \rho\sqrt{\gamma} \\ 0 & \epsilon & 0 \\ \rho^*\sqrt{\gamma} & 0 & \gamma \end{bmatrix} \quad (3)$$

where  $\sigma = E\{|HH|^2\}$ ,  $\epsilon = \frac{E\{|HV|^2\}}{E\{|HH|^2\}}$ , (4)

and  $\gamma = \frac{E\{|VV|^2\}}{E\{|HH|^2\}}$ ,  $\rho\sqrt{\gamma} = \frac{E\{HH VV^*\}}{E\{|HH|^2\}}$

Note that since the target is in a clutter background, the measured target data are modeled (by superposition) as

$$X_{t+c} = X_t + X_c \quad (5)$$

This implies the target-plus-clutter data is also zero mean complex Gaussian with covariance

$$\Sigma_{t+c} = \Sigma_t + \Sigma_c \quad (6)$$

and thus, has the same structure as given in Equation (3) above, with

$$\sigma_{t+c} = \sigma_t + \sigma_c \quad (7)$$

$$\epsilon_{t+c} = \frac{\sigma_t \epsilon_t + \sigma_c \epsilon_c}{\sigma_{t+c}}$$

$$\gamma_{t+c} = \frac{\sigma_t \gamma_t + \sigma_c \gamma_c}{\sigma_{t+c}}$$

$$\rho_{t+c}\sqrt{\gamma_{t+c}} = \frac{\sigma_t \rho_t \sqrt{\gamma_t} + \sigma_c \rho_c \sqrt{\gamma_c}}{\sigma_{t+c}}$$

Also, the input target-to-clutter ratio is defined to be

$$(T/C)_{in} = \frac{\sigma_t}{\sigma_c} \quad (8)$$

#### THE OPTIMAL POLARIMETRIC DETECTOR

Next, in this section of the paper, we derive the optimal polarimetric detector (OPD) for the ideal situation, that is, assuming the parameters ( $\sigma$ ,  $\epsilon$ ,  $\gamma$ ,  $\rho$ ) and the target-to-clutter ratio  $(T/C)_{in}$  are exactly known. The solution we obtain will reveal the structure of the detector which provides the

best possible detection performance achievable under ideal conditions. The performance of this ideal optimal detector provides an upper bound against which other suboptimal polarimetric detection schemes can be compared. For our two-class problem (i.e., target-plus-clutter versus clutter) the likelihood ratio is [3]

$$\frac{f(X | \omega_{t+c})}{f(X | \omega_c)} > T_D \quad \text{say } \omega_{t+c} \quad (9)$$

where we denote the target-plus-clutter class by " $\omega_{t+c}$ " and the clutter only class by " $\omega_c$ ".  $T_D$  is the detection threshold. The solution to this likelihood ratio is easily shown to be a quadratic detector of the form [3]

$$X^\dagger (\Sigma_c^{-1} - \Sigma_{t+c}^{-1}) X + \ln \frac{|\Sigma_c|}{|\Sigma_{t+c}|} > \ln T_D \quad ; \text{ say } \omega_{t+c} \quad (10)$$

Substituting the specific covariance matrices defining our two classes into the above algorithm yields an interesting solution. Rewriting the above solution in a slightly different form, the optimal detector computes the distances to the target-plus-clutter class and the clutter class

$$d_c(X) - d_{t+c}(X) > \ln T_D \quad (11)$$

where  $d_c(X) = X^\dagger \Sigma_c^{-1} X + \ln |\Sigma_c|$  (12)

and  $d_{t+c}(X) = X^\dagger \Sigma_{t+c}^{-1} X + \ln |\Sigma_{t+c}|$  (13)

Evaluating the above distance measures, one obtains an expression for the optimal detection statistic [4]

$$d_i(X) = \frac{|HH|^2}{\sigma_i(1-\rho_i^2)} + \frac{|VV|^2}{\sigma_i(1-\rho_i^2)\gamma_i} + \frac{|HV|^2}{\sigma_i\epsilon_i} \quad (14)$$

$$- \frac{2\rho_i}{\sigma_i(1-\rho_i^2)\sqrt{\gamma_i}} |HH| |VV| \cos(\phi_H - \phi_{VV})$$

$$+ \ln \sigma_i^3 \epsilon_i \gamma_i (1-\rho_i^2) ; i=c, t+c$$

We observe that the fundamental structure of the optimal polarimetric detector makes use of the polarimetric amplitude information ( $|HH|$ ,  $|HV|$ ,  $|VV|$ ) and also the polarimetric phase difference ( $\phi_{HH} - \phi_{VV}$ ) which corresponds to the difference in phase between the HH and VV complex returns. The classifier applies optimal weighting to the observed radar measurement data (as shown in Equation (14) above) prior to making its detection decision.

## THE POLARIMETRIC MATCHED FILTER

In the previous section of the paper we have defined the two-class target detection problem and have derived the detection algorithm which makes optimal use of the observed polarimetric return. This algorithm is optimal in the likelihood ratio sense implying it yields the best possible probability of detection for a given false alarm probability. An alternative approach is to design a linear processor or matched filter which processes the polarimetric return so as to provide maximum target-to-clutter ratio to the radar detector. This algorithm we shall call the polarimetric matched filter (PMF) and is easily derived using the approach given in Reference [6]. For completeness, a brief derivation of this detector is given in the following paragraphs.

Again the assumption is that we have two classes (the target-plus-clutter class and the clutter class) but we now seek the best linear weight vector for processing the polarimetric data vector.

Thus, we seek the linear combination  $y = \mathbf{h}^T \mathbf{x}$  which provides maximum target-to-clutter ratio at the filter output, which is

$$(T/C)_{\text{out}} = \frac{\mathbf{h}^T \Sigma_t \mathbf{h}}{\mathbf{h}^T \Sigma_c \mathbf{h}} \quad (15)$$

We remark that the polarimetric matched filter makes use of the target and clutter covariances  $\Sigma_t$  and  $\Sigma_c$  respectively. This implies a design which is independent of the actual input target-to-clutter ratio, i.e., a constant coefficient filter. It is well known [6] that the optimal weight vector, denoted  $\mathbf{h}^*$ , is obtained as the solution to the generalized eigenvalue problem

$$\Sigma_t \mathbf{h}^* = \lambda^* \Sigma_c \mathbf{h}^* \quad (16)$$

where  $\mathbf{h}^*$  is the eigenvector corresponding to the maximum eigenvalue,  $\lambda^*$ . Also, the maximum eigenvalue  $\lambda^*$  is actually the optimal target-to-clutter ratio out of the filter which is obtained as a result of using the optimal  $\mathbf{h}^*$ . Equivalently, one may solve the following simpler eigenvalue-eigenvector problem to obtain the  $(\lambda^*, \mathbf{h}^*)$  solution

$$\Sigma_c^{-1} \Sigma_t \mathbf{h}^* = \lambda^* \mathbf{h}^* \quad (17)$$

It is more convenient to solve this equivalent eigenvalue problem since the structure of the

matrix  $\Sigma_c^{-1} \Sigma_t$  is simple and easily leads to an exact analytical solution. The three eigenvectors obtained are of the form

$$\mathbf{h}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{h}_2 = \begin{bmatrix} 1 \\ 0 \\ \beta_2 \end{bmatrix}, \quad \mathbf{h}_3 = \begin{bmatrix} 1 \\ 0 \\ \beta_3 \end{bmatrix} \quad (18)$$

where the parameters  $\beta_2$  and  $\beta_3$  are given by the expression

$$\beta_{2,3} = \pm \frac{\left[ 4\gamma_c \gamma_t \rho_t^2 - 4\sqrt{\gamma_c \gamma_t^3} \rho_c \rho_t - 4\sqrt{\gamma_t \gamma_c^3} \rho_c \rho_t + 4\gamma_c \gamma_t \rho_c^2 + \gamma_t^2 - 2\gamma_c \gamma_t + \gamma_c^2 \right]^{1/2} + \gamma_t - \gamma_c}{2\gamma_c \sqrt{\gamma_t \rho_t} - 2\sqrt{\gamma_c \gamma_t} \rho_c} \quad (19)$$

The optimal polarimetric matched filter corresponds to one of the above three solutions and is defined by the maximum of the three eigenvalues  $(\lambda_1, \lambda_2, \lambda_3)$ . Thus the polarimetric matched filter can be one of the three possible linear combinations of the polarimetric measurements, namely

$$\begin{aligned} \text{(i)} \quad y_1 &= HV \\ \text{(ii)} \quad y_2 &= HH + \beta_2 VV \\ \text{(iii)} \quad y_3 &= HH + \beta_3 VV \end{aligned} \quad (20)$$

To gain simple insight into the above solution, we can show that for the special case when  $\gamma_t = \gamma_c = 1$ , the optimal polarization combinations become

$$\begin{aligned} \text{(i)} \quad y_1 &= HV \\ \text{(ii)} \quad y_2 &= HH + VV \\ \text{(iii)} \quad y_3 &= HH - VV \end{aligned} \quad (21)$$

These three solutions correspond to the following simple target-in-clutter situations

- (i) HV is the polarization which provides maximum signal return for a dihedral reflector in a homogeneous clutter background with the dihedral oriented at  $\pm 45^\circ$  relative to the horizontal.
- (ii) HH+VV is the polarization combination which provides maximum signal return for a trihedral reflector in a homogeneous clutter background.
- (iii) HH-VV is the polarization combination which provides maximum signal return for a dihedral reflector in homogeneous clutter with the dihedral oriented horizontally or vertically.

Other alternative approaches may be used in detecting targets in clutter which are independent of the parameters of the target-plus-clutter and clutter classes. These suboptimal detection algorithms make use of lesser amounts of polarimetric information and we will consider two of these methods. The first scheme (used extensively in various radar applications by numerous researchers) processes the complex radar return by computing the polarimetric span according to the relation

$$y = |HH|^2 + 2|HV|^2 + |VV|^2 \quad (22)$$

The span detection statistic makes use of the total power in the polarimetric return and has the property that it is invariant with respect to the polarization basis used by the radar. The span is actually a suboptimal quadratic detector, since it is obtained from the simplified algorithm

$$y = (HH^*, HV^*, VV^*) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} HH \\ HV \\ VV \end{bmatrix} > T_0 \quad (23)$$

The span detector does not make use of the polarimetric phase ( $\phi_{HH} - \phi_{VV}$ ) and utilizes only the polarimetric amplitude information, hence we shall gain some insight from the comparison of performance results for the various algorithms as to the usefulness of polarimetric phase in our target detection application. Finally, we will consider a single polarimetric channel radar (specifically, HH) and will compare the performance of this baseline algorithm to that of the more complex algorithms.

#### PRODUCT TARGET AND CLUTTER MODELS

Next, we present the results of a study of the performance sensitivity of these polarimetric detectors under the assumption that the target and clutter models are characterized as having a product model structure. Until now we have assumed a homogeneous clutter background and each clutter pixel in the scene had the same average polarimetric power and covariance between the polarimetric returns. Also, we assumed the target-plus-clutter samples to be from a single Gaussian PDF with a constant average power and covariance. It is more reasonable to assume the clutter background to be spatially nonhomogeneous and the target return to have an aspect angle variability. To this end, we postulate random polarimetric target and clutter models consistent with these more realistic assumptions. Thus, to include the effects of spatial variability of clutter and aspect variability of targets we postulate random polarimetric target and clutter models having the product-model structure. The motivating idea behind this study is to evaluate the effects of spatial variability of clutter and aspect angle variability of targets on the performance of the optimal polarimetric detector, the polarimetric matched filter, and our other simpler detectors. To compare these detectors, we also need an optimal likelihood ratio detector for our more realistic product models of targets and clutter. Thus, we also derive the exact PDF for the product model polarimetric feature vectors and implement the likelihood ratio detector for the product model problem.

Since we are interested in a product model for both targets and clutter, we take the model to be of the form

$$y = \sqrt{\sigma} \underline{x} \quad (24)$$

where  $\sqrt{\sigma}$  represents an arbitrary scale factor. Our basic assumption is that the feature vectors  $\underline{x}$  have a specified covariance matrix  $\Sigma$  and the vec-

tors  $\underline{x}$  are scaled according to some random variable  $\sqrt{\sigma}$ . This comprises our product model for polarimetric data measurements and represents a simple extension of the single polarimetric channel product models of targets and clutter derived in [1]. Determining the PDF of random vector  $y$  is straightforward and proceeds as follows. For a given value " $\sigma$ ", we have

$$E\{y/\sigma\} = \sqrt{\sigma} E\{\underline{x}\} = 0 \quad (25)$$

$$\text{COV}\{y/\sigma\} = \sigma \Sigma \quad (26)$$

The conditional PDF of random vector  $y$  is also complex Gaussian

$$f(y/\sigma) = \frac{1}{\pi^n \sigma^n |\Sigma|} \exp \left\{ -\frac{y^\dagger \Sigma^{-1} y}{\sigma} \right\} \quad (27)$$

Next, we evaluate the unconditional PDF of random vector  $y$  which is obtained from the integral

$$f(y) = \int_0^\infty f(y/\sigma) p(\sigma) d\sigma \quad (28)$$

where  $p(\sigma)$  is the PDF of the scalar product multiplier. We find it convenient to assume a Gamma (or chi-square) distributed cross-section model with density

$$p(\sigma) = \frac{1}{\sigma} \left( \frac{\sigma}{\bar{\sigma}} \right)^{v-1} \frac{1}{\Gamma(v)} \exp \left\{ -\frac{\sigma}{\bar{\sigma}} \right\} \quad (29)$$

As in References [7-9], we adopt the above two-parameter cross-section model where  $v$  is the order parameter and  $\bar{\sigma}$  is related to the mean radar cross-section. This model was shown to yield the K-distribution for a single polarimetric channel ground clutter and sea clutter. We have shown that this distribution is a reasonable model for both radar ground clutter and targets similar to that data collected using the HOWLS [1] radar. Thus, it is reasonable to apply this cross-section model to the polarimetric feature vector problem and we will show that this leads to a generalized K-distribution for the PDF of random vector,  $y$ . Substituting Equations (27) and (29) into Equation (28), we obtain the result

$$f(y) = \frac{2}{\pi^n \bar{\sigma}^v \Gamma(v) |\Sigma|} \left( \frac{y^\dagger \Sigma^{-1} y}{\bar{\sigma}} \right)^{\frac{n-v}{2}} K_{n-v} \left( 2 \sqrt{\frac{y^\dagger \Sigma^{-1} y}{\bar{\sigma}}} \right) \quad (30)$$

Given the exact PDF for the product model characterization of targets and clutter, we next obtain the corresponding optimal log-likelihood ratio detector. Omitting the unnecessary details, we obtain the distance measures  $D_{t+c}(y)$  and  $D_c(y)$

$$D_i(\underline{y}) = (v_i - n) \ln(d_i^2 / 2\bar{\sigma}_i) + \ln K_{v_i - n} \left( 2 \sqrt{\frac{d_i^2}{2\bar{\sigma}_i}} \right) - \ln \Gamma(v_i) - \ln |\Sigma_i| - v_i \ln \bar{\sigma}_i \quad (31)$$

$$\text{where } d_i^2 = \underline{y}^T \Sigma_i^{-1} \underline{y}, \quad i = c, t+c$$

The optimal polarimetric detector defined above is of the same form as derived previously in Equation (11), i.e., the detection statistic is computed and the decision is

$$D_c(\underline{y}) - D_{t+c}(\underline{y}) > \ln T_D; \text{ say "}\omega_{t+c}\text{"} \quad (32)$$

We use this detector in the ideal situation where the parameters ( $\sigma$ ,  $\epsilon$ ,  $\gamma$ ,  $\rho$ ) and target-to-clutter ratio are exactly known as are the parameters ( $v$ ,  $\bar{\sigma}$ ) also known. The performance of this detector provides an upper bound against which we can compare the performance of our other detectors. In this way, we may judge the relative degradation in performance which occurs when the detectors are designed for some nominal target and clutter parameters but tested against product model input data.

#### SENSITIVITY ANALYSIS OF POLARIMETRIC ALGORITHMS

We are interested in evaluating the performance of the polarimetric detectors under the assumption that they have been designed for nominal Gaussian target and clutter statistics. The actual test inputs will then be assumed to have our product model structure and the sensitivity of the detectors to the effects of clutter spatial variability and target aspect variability will be determined. A brief summary of the analysis of the sensitivity of the polarimetric detectors is given in the following paragraphs. For the optimal polarimetric detector, we write

$$\underline{y} = \underline{X}^T (\Sigma_c^{-1} - \Sigma_{t+c}^{-1}) \underline{X} + C \quad (33)$$

where

$$C = \ln \frac{|\Sigma_c|}{|\Sigma_{t+c}|} - \ln T_D \quad (34)$$

Taking the approach of References [10,11], we evaluate the conditional characteristic function of random variable,  $y$ , which is

$$\phi_{y/\sigma}(j\omega) = \int \dots \int \exp \left\{ j\omega \left( \underline{X}^T [\Sigma_c^{-1} - \Sigma_{t+c}^{-1}] \underline{X} + C \right) \right\} \frac{\exp \left\{ \frac{-\underline{X}^T \Sigma^{-1} \underline{X}}{\sigma} \right\}}{\pi^n \sigma^n |\Sigma|} d\underline{X} \quad (35)$$

The above expression implies we have designed the detector using nominal  $\Sigma_{t+c}$  and  $\Sigma_c$  for our target-plus-clutter and clutter classes but are testing the algorithm with measurement data having a product model structure by appropriately selecting  $\Sigma$  and  $\sigma$ . For now, however, we assume a given " $\sigma$ " and evaluate the exact characteristic function to be of the form

$$\phi_{y/\sigma}(j\omega) = e^{j\omega C} \prod_{i=1}^3 \frac{1}{(1 - j2\sigma\lambda_i\omega)} \quad (36)$$

where the eigenvalues ( $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ) are obtained from the simultaneous diagonalization of the matrices

$$\Sigma_c^{-1} - \Sigma_{t+c}^{-1} \text{ and } \Sigma^{-1} \quad (37)$$

The eigenvalues were obtained as analytical closed-form expressions using MAXIMA [12]. The probability density  $f_{y/\sigma}(y)$  is obtained by expanding  $\phi_{y/\sigma}(j\omega)$  in partial fractions and taking the inverse transform. Integrating  $f_{y/\sigma}(y)$  yields

$$P_{D/FA}(\sigma) = \sum_{i=1}^3 A_i P_i(\sigma) \quad (38)$$

where

$$P_i(\sigma) = 1 - \exp \left\{ \frac{C}{2\sigma\lambda_i} \right\}; \quad \lambda_i > 0, C < 0 \quad (39)$$

$$P_i(\sigma) = 0; \quad \lambda_i > 0, C > 0$$

$$P_i(\sigma) = \exp \left\{ \frac{C}{2\sigma\lambda_i} \right\}; \quad \lambda_i < 0, C > 0$$

$$P_i(\sigma) = 1; \quad \lambda_i < 0, C < 0$$

When the test inputs have the product model characterized by the random variable  $\sigma$ , we average the detection probability with respect to  $\sigma$  and obtain

$$\bar{P}_{D/FA} = E_{\sigma}\{P_{D/FA}(\sigma)\} = \sum_{i=1}^3 A_i E\{P_i(\sigma)\} \quad (40)$$

where

$$E_{\sigma}\{P_i(\sigma)\} = 1 - \left[ \frac{-C\bar{\sigma}}{2\lambda_i} \right]^{\frac{v}{2}} \frac{K_v \left( 2 \sqrt{\frac{-C}{2\lambda_i\bar{\sigma}}} \right)}{\bar{\sigma}^v \Gamma(v)}; \quad \lambda_i > 0, C < 0 \quad (41)$$

$$E_{\sigma}\{P_i(\sigma)\} = 0; \quad \lambda_i > 0, C > 0$$

$$E_{\sigma}\{P_i(\sigma)\} = \left[ \frac{-C\bar{\sigma}}{2\lambda_i} \right]^{\frac{v}{2}} \frac{K_v \left( 2 \sqrt{\frac{-C}{2\lambda_i\bar{\sigma}}} \right)}{\bar{\sigma}^v \Gamma(v)}; \quad \lambda_i < 0, C > 0$$

$$E_{\sigma}\{P_i(\sigma)\} = 1; \quad \lambda_i < 0, C < 0$$

The exact  $P_{D/FA}$  performance of the OPD for homogeneous target and clutter models is obtained from Eq. (39) above by taking the random multiplier  $\sigma=1$ . Thus the exact solution for the detection performance of the OPD involves calculation of the three eigenvalues  $\lambda_1, \lambda_2, \lambda_3$  from simultaneous diagonalization of the covariance matrices of Equation (37) above.

We are interested in evaluating the performance of the OPD when two or more independent measurements of the polarimetric data,  $X$ , are observed and processed in an optimal manner. The extension of the theory to the multi-look case is summarized in the following paragraphs. The assumption we make is that each observed polarimetric measurement vector from class " $\omega_c$ " has the same statistics ( $\underline{Q}, \Sigma_c$ ) and each polarimetric measurement vector from class " $\omega_{t+c}$ " has the same statistics ( $\underline{Q}, \Sigma_{t+c}$ ). With these assumptions, it is easy to show that the likelihood ratio test for "m" independent observations is comprised of sequential processing of each observed vector  $X_i, i=1,2,\dots,m$  in the single-look quadratic classifier [13]. The single-look detection statistics,  $y_i$ , are then summed and compared to the detection threshold,  $T_D$ . Finally, since the characteristic function of a sum of independent random variables is the product of the individual characteristic functions, we obtain for the m-look case

$$\phi_{y/\sigma}^{(m)}(jw) = e^{jwC} \prod_{i=1}^3 \frac{1}{(1-j2\sigma\lambda_i w)^m} \quad (42)$$

From this, one may easily obtain the exact formulas for detection and false alarm probabilities. The solution is lengthy and only the final results will be given here.

$$P_{D/FA}^{(m)}(\sigma) = \sum_{i=1}^3 \sum_{\ell=1}^m A_{i\ell}^{(m)} p_{i\ell}^{(m)} \quad (43)$$

where

$$p_{i\ell}^{(m)} = G_{i\ell}(\sigma) \quad ; \quad \lambda_i > 0, c < 0 \quad (44)$$

$$p_{i\ell}^{(m)} = 1 \quad ; \quad \lambda_i > 0, c > 0$$

$$p_{i\ell}^{(m)} = 1 - G_{i\ell}(\sigma) \quad ; \quad \lambda_i < 0, c > 0$$

$$p_{i\ell}^{(m)} = 0 \quad ; \quad \lambda_i < 0, c < 0$$

where

$$G_{i\ell}(\sigma) = \int_{-\frac{C}{\sigma\lambda_i}}^{\infty} \frac{x^{\ell-1} e^{-x/2}}{2^{\ell}(\ell-1)!} dx \quad (45)$$

and

$$A_{i\ell} = \frac{(-2\lambda_i)^{\ell}}{(m-\ell)! \prod_{k=1}^3 (-2\lambda_k)^m} \sum_{n=0}^{m-\ell} \binom{m-\ell}{n} \quad (46)$$

$$\frac{[(-m-n+1)(-m-n+2)\dots(-m)][(-2m+\ell+n+1)\dots(-m)]}{\left[\frac{1}{2\lambda_i} - \frac{1}{2\lambda_{i1}}\right]^{-m-n} \left[\frac{1}{2\lambda_i} - \frac{1}{2\lambda_{i2}}\right]^{-2m+\ell+n}}$$

where  $i_1 = \text{modulo}_3(i) + 1$

$i_2 = \text{modulo}_3(i+1) + 1$

Finally, when the test inputs have the product multiplier,  $\sigma$ , which is characterized by the Gamma distribution of Equation (29), we take the expectation with respect to this variable and obtain

$$E_{\sigma}\{P_{D/FA}^{(m)}(\sigma)\} = \sum_{i=1}^3 \sum_{\ell=1}^m A_{i\ell} E_{\sigma}\{p_{i\ell}^{(m)}\} \quad (47)$$

where

$$E_{\sigma}\{p_{i\ell}^{(m)}\} = E_{\sigma}\{G_{i\ell}(\sigma)\} \quad ; \quad \lambda_i > 0, c < 0 \quad (48)$$

$$E_{\sigma}\{p_{i\ell}^{(m)}\} = 1 \quad ; \quad \lambda_i > 0, c > 0$$

$$E_{\sigma}\{p_{i\ell}^{(m)}\} = 1 - E_{\sigma}\{G_{i\ell}(\sigma)\} \quad ; \quad \lambda_i < 0, c > 0$$

$$E_{\sigma}\{p_{i\ell}^{(m)}\} = 0 \quad ; \quad \lambda_i < 0, c < 0$$

and  $E_{\sigma}\{G_{i\ell}(\sigma)\} =$

$$\frac{1}{\sigma^v \Gamma(v)} \sum_{k=0}^{\ell-1} \frac{\left(-\frac{C}{\lambda_i}\right)^{v+k} \left(\frac{\sigma}{2}\right)^{v-k}}{2^{k-1} k!} K_{v-k} \left(2\sqrt{\frac{-C}{2\sigma\lambda_i}}\right) \quad (49)$$

Analysis of the matched filter algorithm is simpler due to the fact that the algorithm is linear. We briefly summarize the solution for the multi-look case. The output of the filter is a complex Gaussian comprised of the optimal weighted sum of the HH, HV, and VV data. This signal is noncoherently detected and summed prior to being compared with the detection threshold,  $T_D$ . Mathematically, the procedure is represented as

$$y = \sum_{k=1}^m |h^{\dagger} X_k|^2 > T_D \quad ; \quad \text{say } "\omega_{t+c}" \quad (50)$$



Since random variable,  $y$ , is chi-square (the sum of  $m$ -independent exponential variables) we need only compute  $E\{|\underline{h}^T \underline{X}|^2\}$  for class " $\omega_{t+c}$ " and for class " $\omega_c$ " in order to determine detection performance of the algorithm. We obtain

$$E\{|\underline{h}^T \underline{X}|^2\} = \sigma d^2(\underline{h}) \quad (51)$$

where  $d^2(\underline{h}) = \underline{h}^T \Sigma \underline{h}$

and  $\sigma$  is the product multiplier. The conditional detection and false alarm probabilities, for a given value of the multiplier,  $\sigma$ , are

$$P_{D/FA}^{(m)}(\sigma) = \sum_{k=0}^{m-1} \frac{\left(\frac{T_D}{\sigma d^2(\underline{h})}\right)^k}{k!} \exp\left\{-\frac{T_D}{\sigma d^2(\underline{h})}\right\} \quad (52)$$

Again, the homogeneous target and clutter case are obtained by taking  $\sigma=1$  in the above expression. When the product multiplier is modeled as the Gamma random variable with parameters  $\{(v_i, \sigma_i) \mid i=t+c, c\}$  we obtain the average detection performance to be

$$E\{P_{D/FA}(\sigma)\} = \frac{2}{\sigma \Gamma(v)} \sum_{k=0}^{m-1} \frac{(T_D/d^2(\underline{h}))^k}{k!} \left(\frac{T_D \bar{\sigma}}{d^2(\underline{h})}\right)^{\frac{v-k}{2}} K_{v-k}\left(2 \frac{T_D}{d^2(\underline{h}) \bar{\sigma}}\right) \quad (53)$$

We have two suboptimal polarimetric detectors which are under investigation and these use simpler detection statistics based on the polarimetric span ( $|\underline{H}\underline{H}|^2 + 2|\underline{H}\underline{V}|^2 + |\underline{V}\underline{V}|^2$ ) and the single channel,  $|\underline{H}\underline{H}|^2$ . Analysis of both these algorithms is a straightforward modification of the above results. For example, the single  $|\underline{H}\underline{H}|$  channel detection results are obtained by substituting for the matched filter

$$\underline{h}^T = (1 \ 0 \ 0) \quad (54)$$

and evaluating detection performance using Equations (53), (52).

Similarly, the detection statistic based on the polarimetric span is easily evaluated by noting that this detector is quadratic and of the form

$$y = \underline{X}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{X} + C > 0 ; \text{ say } "\omega_{t+c}" \quad (55)$$

Thus, we substitute Equation (55) into Equation (35) to obtain the characteristic function  $\phi_{y/\sigma}(j\omega)$  and use our previously developed solution to evaluate detection performance for this algorithm.

## ALGORITHM PERFORMANCE PREDICTIONS

The performance results presented in this section of the paper are based on polarimetric measurement data of typical ground targets and meadow clutter. The polarimetric parameters of these targets and clutter are tabulated below (see Table 1). Detection performance results presented in the following paragraphs are for target 1 versus clutter. Target discrimination results presented are for target 1, an armored target, versus target 2, a truck.

### 1. OPD Performance Results

We have compared the performance of the OPD with both span and single channel  $|\underline{H}\underline{H}|^2$  processing. Note that since OPD and span processing require two pulses per look, we compare HH processing also with two pulses per look. For the latter case, we

TABLE 1

POLARIMETRIC PARAMETERS OF TARGETS AND CLUTTER

	$\sigma(m^2)$	$\epsilon$	$\gamma$	$\rho\sqrt{\gamma}$
TARGET 1	58.5	0.19	1.0	0.28
TARGET 2	618.3	0.02	1.1	0.83
CLUTTER	4.75	0.18	1.6	0.63

assume two independent HH samples per look obtainable using pulse-to-pulse frequency diversity. For single-look processing with homogeneous target and clutter models, the curves of Figure 1 indicate that HH processing outperforms span processing even though the span detector uses all three polarimetric amplitudes. The OPD, of course, provides the best performance since it utilizes all the polarimetric information in an optimally weighted fashion. We remark, however, that the performance improvement of the OPD is not really significant relative to HH processing. Furthermore, to achieve this improvement in detection performance requires exact knowledge of the target-to-clutter ratio and also the target and clutter covariance statistics since the optimal weighting coefficients are computed from this information. Since these target and clutter statistics would be difficult to predict a priori, implementing the OPD in a real system would be difficult. For these reasons, the OPD does not appear to have a significant advantage over HH processing.

The contribution of the polarimetric phase term,  $|\underline{H}\underline{H}| |\underline{V}\underline{V}| \cos(\phi_{HH} - \phi_{VV})$ , in this target detection application does not appear to be significant. Specifically, the distance measures of Equation 14 are dominated by the radar cross-section terms ( $|\underline{H}\underline{H}|^2, |\underline{V}\underline{V}|^2, |\underline{H}\underline{V}|^2$ ). To further support this claim, we have evaluated the detection performance using amplitude-normalized feature vectors. The optimal processor of normalized data (OPDN) derived in Reference [14], provides the best possible performance for normalized

Gaussian feature vectors. The theoretically optimal performance for the normalized data is shown in the curves of Figure 2. A comparison of the performance of the optimal processor for normalized data (Figure 2) with that of the OPD which processes unnormalized data (Figure 1) clearly shows that it is the polarimetric amplitude information which achieves the good detection performance results for the OPD.

Figure 3 summarizes the performance predictions for the 6 dB target-to-clutter case using multi-look processing. From the curves, it is again clear that HH detection performance is superior to detection using the span statistic. An optimally weighted combination of the  $|HH|^2$ ,  $|VV|^2$  and  $|HV|^2$  amplitudes might improve performance of the span detector somewhat, however, the upper bound obtained by the OPD cannot be exceeded and HH processing is not significantly degraded relative to this bound. We remark that the results of Figure 3 which correspond to multi-look processing of statistically independent PSM samples are very optimistic due to the idealized homogeneous target and clutter models used. With more realistic product model representations of targets and clutter, we will obtain performance results for HH processing which are consistent with those achieved using the HOWLS [1] radar data.

The curves of Figures 4 and 5 show more realistic algorithm performance predictions based on the product target and clutter models. Note that in these figures, we show the performance of the OPD which was designed for homogeneous target and clutter models but tested against nonhomogeneous product target and clutter model inputs. These results were obtained using the analysis presented previously. Comparing the results of Figure 4 with Figure 1 shows the deleterious effect of nonhomogeneous target and clutter on the performance of all the algorithms. For example, for  $P_{FA} = 0.001$  the detection performance of the OPD with 10 dB target-to-clutter has been reduced from 90 percent to less than 70 percent. Similar reductions can be observed for the other algorithms and at other target-to-clutter ratios. Note also the performance improvement achieved through multi-look processing is considerably reduced when the more realistic target and clutter models are used (compare Figure 3 with Figure 5). Thus, the benefits of frequency averaging of independent PSM samples are reduced due to nonhomogeneity of the target and clutter models. These observations are consistent with the results obtained previously using HOWLS data [1]. Also, we remark that the improved performance of the OPD relative to HH processing is not as significant since the OPD detector has been designed to be optimal for homogeneous target and clutter models.

From our previous studies [1], we have shown the nonhomogeneity of ground clutter and aspect angle variability of targets to be dominant factors in the reduction of detection performance, that is, the sensitivity to the target and clutter st. dev. parameters is quite severe. To verify this effect also applies to our polarimetric detection schemes

we have evaluated the performance of the OPD over a reasonable range of  $\sigma_c$  (1, 1.5, 2, 2.5, and 3 dB). Figure 6 shows the OPD performance for single-look and 4-look processing. Note that the top curves correspond to the homogeneous clutter model and are included as an upper bound on performance. From the curves, it is clear that detection performance is severely affected by the nonhomogeneity of clutter. We have also evaluated the performance of the OPD over a reasonable range of  $\sigma_t$  (1, 2, 3, and 4 dB). Figure 7 shows the corresponding performance results. From these curves, it is seen that the single-look results are less affected by the change in  $\sigma_t$  than the 4-look case, but there is, in general, a strong dependence on  $\sigma_t$ .

We are interested in determining the performance of the LRT test which defines the optimal processing of product-model targets and clutter. This algorithm was defined in Equation (31). Since the OPD exhibited degraded performance when designed for homogeneous models, but tested with nonhomogeneous models, we would like to have a performance comparison of the OPD with that of the LRT algorithm. Our studies indicate that over the range of parameter variations of interest, the OPD performs almost as well as the LRT test.

## 2. PMF Detection Performance Results

A summary of our polarimetric matched filter studies is presented next. We have designed the PMF based on the target and clutter covariances specified earlier. Evaluating the eigenvalues and eigenvectors of the matrix  $\Sigma_c^{-1} \Sigma_{t+c}$ , we have the following solutions

$$\begin{aligned} \text{(i)} \quad \lambda_1 = 12.78 \leftrightarrow \underline{h}_1 &= \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} \\ \text{(ii)} \quad \lambda_2 = 7.54 \leftrightarrow \underline{h}_2 &= \begin{matrix} 1 \\ 0 \\ 3.6 \end{matrix} \\ \text{(iii)} \quad \lambda_3 = 15.58 \leftrightarrow \underline{h}_3 &= \begin{matrix} 1 \\ 0 \\ -0.5 \end{matrix} \end{aligned}$$

The best PMF is, therefore, specified by the solution (iii) above. We have compared the detection performance of this PMF with that of the single channel ( $|HH|^2$ ) detector. One of our objectives is to make a direct comparison of the PMF with the results of the HOWLS radar [1], so for these studies we have used the product target and clutter models with parameters  $\sigma_t = 3$  dB,  $\sigma_c = 2$  dB and  $(T/C) = 6$  dB. Figure 8 summarizes the detection results obtained. Equations (52) and (53) were used to obtain these theoretical predictions. Referring to the curves of Figure 8, we show the detection performance of the PMF with 1, 2, 4, 8, and 16 independent polarimetric samples processed.

Since these polarimetric samples require transmitting 2, 4, 8, 16, and 32 radar pulses, we show the comparison with  $|HH|^2$  processing with these same numbers of transmitted pulses. Our results are summarized as follows. The PMF performance (with an equivalent number of transmitted pulses) does not perform as well as  $|HH|^2$  processing until we process about 8 independent fully polarimetric measurements. With 8 independent looks (16 pulses transmitted) the two algorithms obtain essentially the same performance. Note that the performance predictions with 16 pulses transmitted agree closely with the HOWLS measurements (i.e.,  $P_D \sim 50$  percent and  $P_{FA} \sim 10^{-3}$ ). As more independent looks are processed, the PMF begins to outperform the  $|HH|^2$  detector.

We have also made a performance comparison of the best PMF design with a detector using a circular transmit, circular receive (LL) system. The single channel  $|LL|$  detection results were obtained by substituting for the matched filter  $h^\dagger = (0.5, j, 0.5)$ . It has been reported that this scheme achieves better target-to-clutter ratio than the linear transmit, linear receive (HH) system. This is because the even bounce (LL) clutter return is less than the HH clutter return whereas the even bounce (LL) target return is about the same as the HH target return. Note that a single LL measurement also requires only 1 pulse transmission. We observe that the even channel (LL) detection performance is slightly better than the HH detection performance. All three detectors are essentially equivalent in performance with 16 pulses transmitted.

### 3. Discrimination Performance of the OPD

Once an object has been detected, we are interested in discriminating between different target types (e.g., tank versus truck) based upon differences in polarimetric information observed by the radar. Clearly, the polarimetric measurement data for targets may contain information as to the type of target being observed and this information may be used by an optimal polarimetric classifier to achieve discrimination. In our studies, we use feature vectors which are normalized by the Euclidean norm. This has the advantage of removing the product scale factor from the data and the classifier design becomes independent of absolute radar cross-section. Only the relative amplitude differences between HH, HV, and VV channels and the polarimetric phase  $\phi_{HH} - \phi_{VV}$  are used

to discriminate between target types. The results of this study are summarized in Table 2 which shows the average probability of classification error for the two targets versus the number of independent polarimetric measurements processed for target-to-clutter ratios of 0, 3, 6, and 10 dB for target 1 versus clutter. The corresponding target-to-clutter ratio for target 2 versus clutter is 10 dB higher due to the larger radar cross-section of target 2 (see Table 1).

The use of polarimetric information in discriminating between target types appears to be promising on the basis of the results shown in Table 2. However, to achieve reliable performance requires

TABLE 2  
PROBABILITY OF CLASSIFICATION ERROR (%)

T/C Ratio	Number of Looks			
	1	2	4	8
10 dB	24.6	17.6	10.6	5.2
6 dB	26.0	19.2	12.0	6.2
3 dB	27.8	21.0	14.0	7.4
0 dB	30.2	24.3	17.1	10.0

multi-look processing with reasonably high (6-10 dB) target-to-clutter ratios. We remark, however, that good discrimination can only be achieved for targets exhibiting discernable differences in polarization characteristics.

### SUMMARY AND CONCLUSIONS

This paper summarizes a study of target detection algorithms which use polarimetric radar data. A model which accounts for the spatial nonhomogeneity of ground clutter and the aspect angle variability of targets has been developed and the performance of various algorithms evaluated. Even when processed in an optimal fashion, the additional information provided by the full PSM measurement does not appear to aid significantly in target detection. A radar which transmits and receives a single polarization (e.g., HH or LL) would obtain almost as good performance as one which measures the full PSM. To achieve the additional performance improvement from the OPD, one must have exact knowledge of the target and clutter covariances including the target-to-clutter ratio. When these covariances are not known, the single polarimetric channel detector would provide the best performance. For these reasons, the utilization of independent multilook single polarization algorithms appears to be the best approach. If a single polarization is used, our studies have shown that LL (even bounce) circular polarization provides slightly better performance than HH (linear) polarization. However, the clutter data base used in these studies was limited, and further study of this problem using various types of clutter (for example, snow clutter) is necessary. In addition, other detection algorithms which process LL, LR dual channel receive data should also be investigated. Once the target is detected, however, information contained in the PSM may be useful in discriminating between target types (e.g., tank versus truck). Our preliminary results indicate that many independent looks at the target are required and the target-to-clutter ratio must be fairly high for these algorithms to be effective. This problem, using a larger set of targets, requires further study. Also, a more realistic

target model, which will be investigated in the future, is one which would characterize a target over different aspect angle sectors with different covariance matrices since some targets may have different polarimetric properties at different aspect angles.

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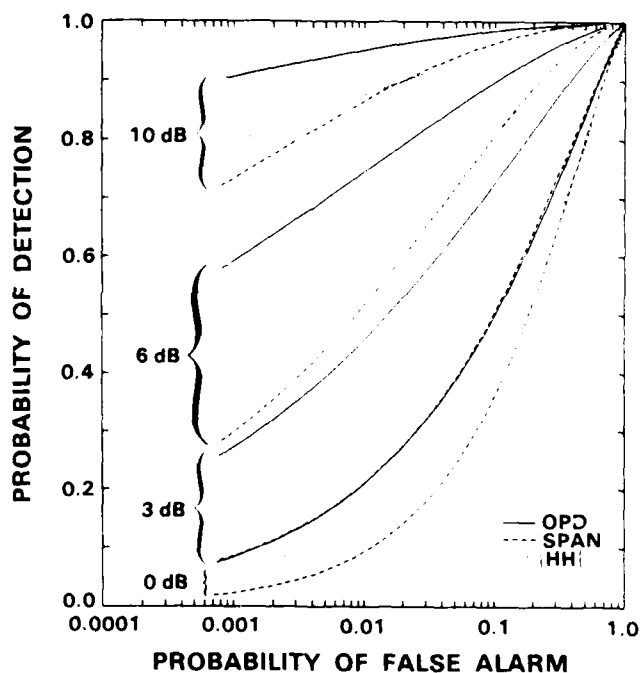


Figure 1. Algorithm performance comparison vs T/C ratio (single-look, homogeneous models).

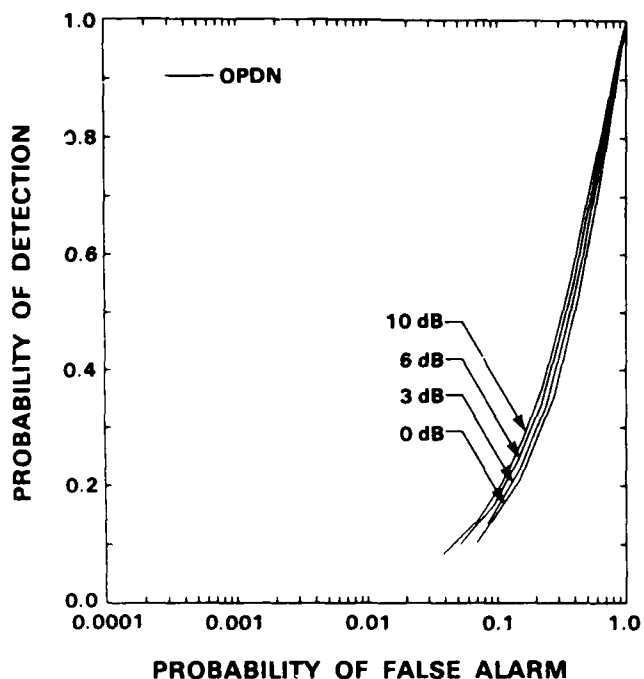


Figure 2. Performance of optimal normalized polarimetric detector vs T/C ratio (single-look, homogeneous models).

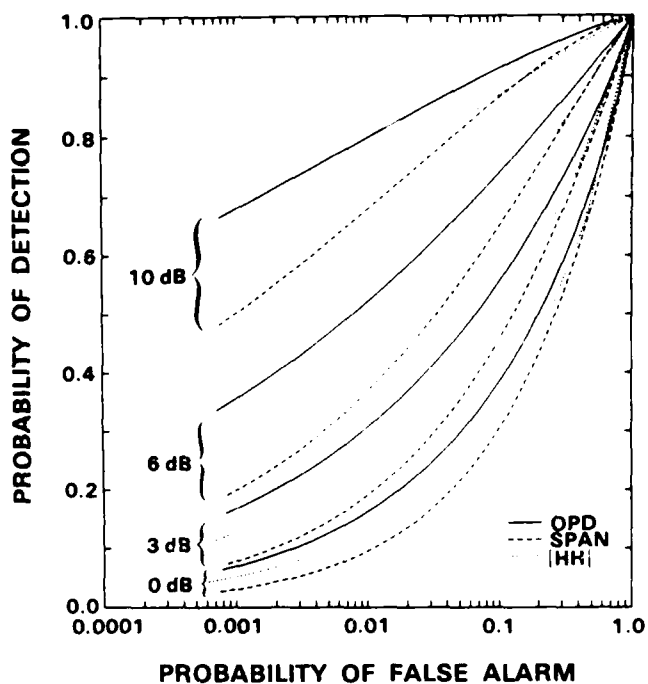


Figure 4. Algorithm performance comparison vs T/C ratio (single-look product models ( $\sigma_t = 3$  dB,  $\sigma_c = 1.5$  dB)).

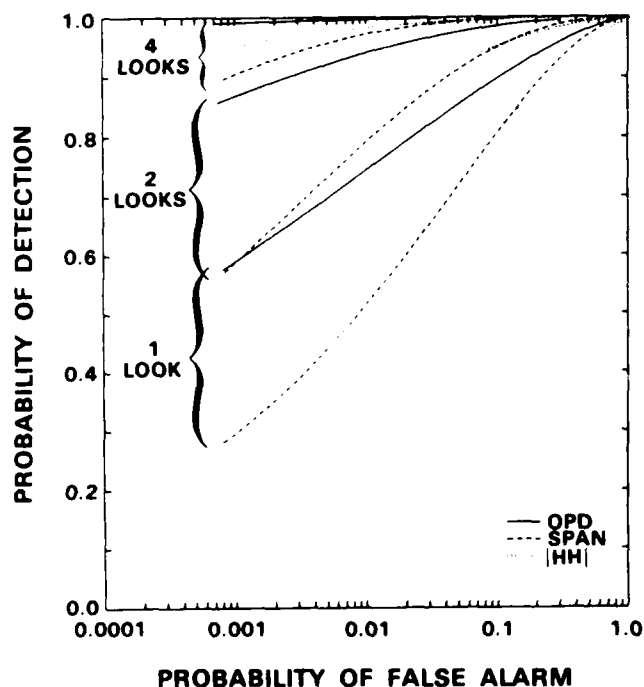


Figure 3. Algorithm performance comparison vs N looks (T/C = 6 dB, homogeneous models).

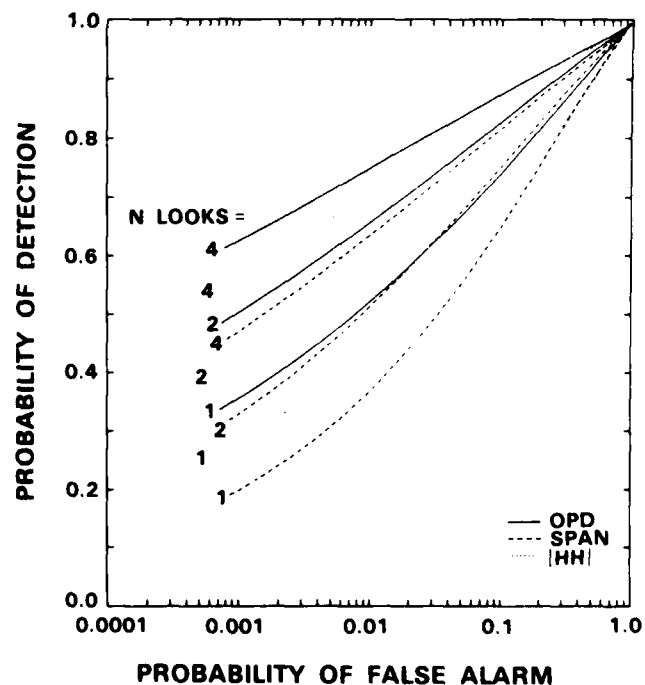


Figure 5. Algorithm performance comparison vs N looks (T/C = 6 dB, product models ( $\sigma_t = 3$ ,  $\sigma_c = 1.5$  dB)).

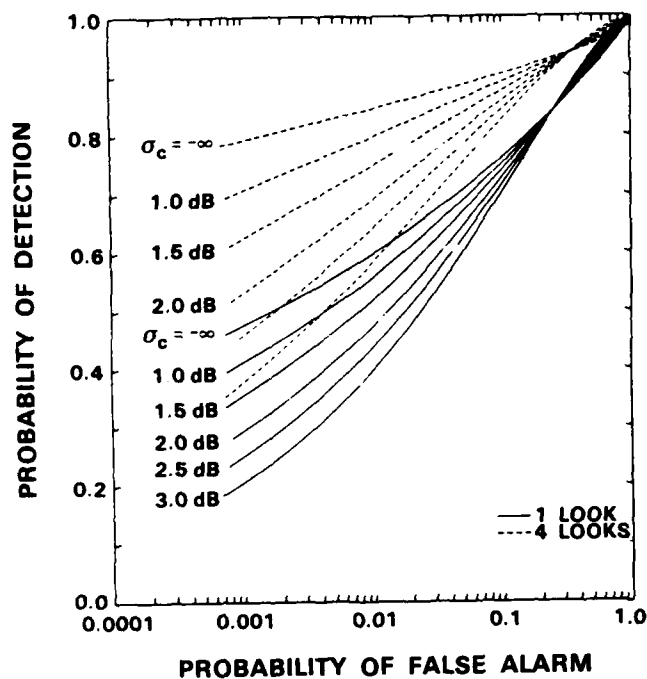


Figure 6. Sensitivity of OPD to clutter St. Dev. (T/C = 6 dB, product model ( $\sigma_t = 3$  dB)).

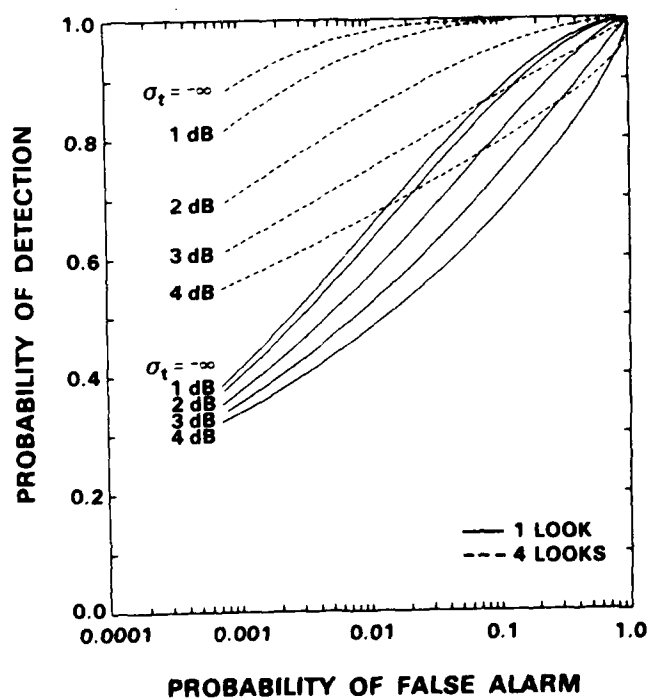


Figure 7. Sensitivity of OPD to target St. Dev. (T/C = 6 dB, product model ( $\sigma_c = 1.5$  dB)).

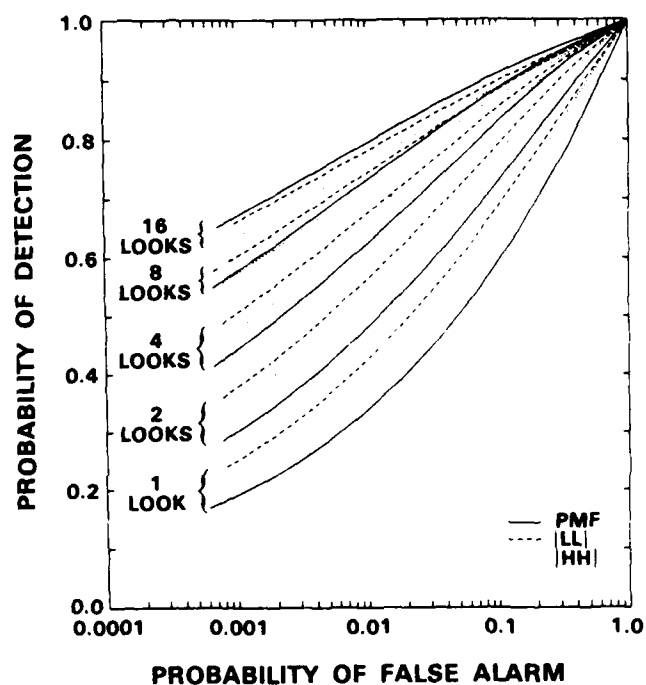


Figure 8. Algorithm performance comparison (T/C = 6 dB, product models ( $\sigma_t = 3$  dB,  $\sigma_c = 2$  dB)).